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Short communication

A numerical analysis for cracks emanating from a surface semi-spherical cavity in an infinite elastic body by FRANC3D

Xinke Xiao, Xiangqiao Yan *

Research Laboratory on Composite Materials, Harbin Institute of Technology, Harbin, 150001, PR China

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Abstract

This note is specifically concerned with cracks emanating from a surface semi-spherical cavity in an infinite body (see Fig. 1) by using the boundary element software FRANC3D developed by a fracture mechanics investigation group of Cornell University. The numerical results can reveal the effect of the geometry of the surface cavity on the stress intensity factors.
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Keywords: Surface cavity; Surface crack; Stress intensity factor

1. Introduction

Due to the stress concentration effect around the hole, cracks are likely to initiate at the hole under the action of fatigue loading. Consequently, a number of papers dealing with hole edge crack problems are available. Bowie [1] gave solutions of a circular hole with a single edge crack and a pair of symmetrical edge cracks in a plate under tension. Newman [2] by means of the boundary collocation method, and Murakami [3] by using the body force method performed analysis of the tension problem for an elliptical hole with symmetrical edge cracks. Tweed and Rooke [4] used the Mellin transform technique to make analysis of biaxial tensions for a branching crack emanating circle hole. Isida and Nakamura [5] made an analysis of a slant crack emanating from an elliptical hole under uniaxial tension and shear at infinity by using the body force method. Recently, Yan [6–9] used the displacement discontinuity method with crack-tip elements to study cracks from a hole in finite plate. For cracks emanating from a circular hole in rectangular plate in tension [9], especially, it was found that the circular hole has the shielding effect on the cracks and the amplifying effect on the SIFs, which depends largely on the ratio of the crack size to the hole size. This finding motivates us to pay attention to cracks emanation a surface spherical cavity in an infinite body (see Fig. 1) by using the boundary element software FRANC3D developed by a fracture mechanics investigation group of Cornell University (CFG). By comparing

* Corresponding author.
E-mail address: yanxiangqiao@hotmail.com (X. Yan).

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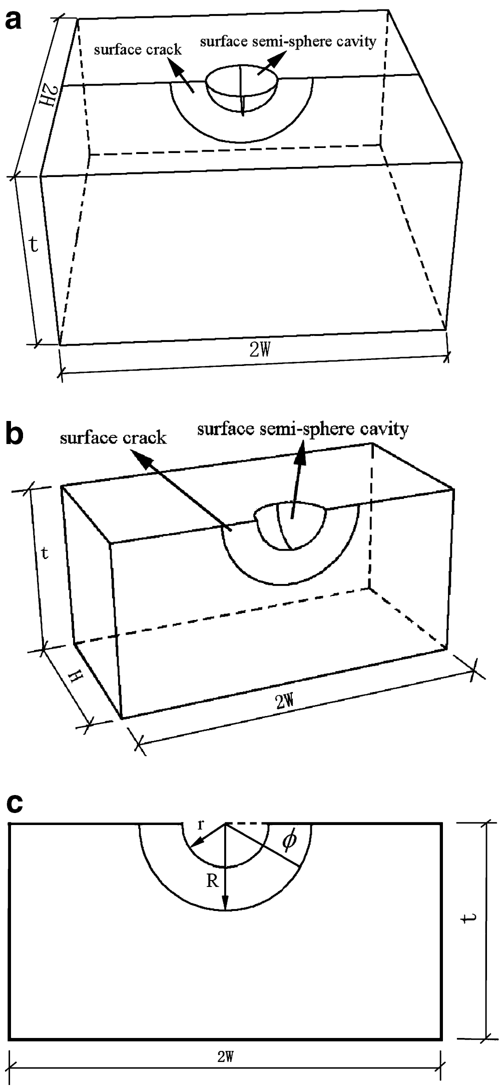

Fig. 1. Schematic of cracks emanating from a surface semi-spherical cavity in infinite elastic body: (a) total view, (b) local view (a half of Fig. (a)) and (c) the symmetry plane in which the crack surface occurs.

the numerical results obtained in this note with those reported in the literature for the surface semi-circular crack, the effect of the geometry of the surface cavity on the stress intensity factors (SIFs) can be revealed.

By the way, the detail of the FRANC3D and the relevant investigations (for example, Refs. [10–12]) can be seen at www.cfg.cornell.edu. For surface crack problems, some typical results are reported by Newman and his coworkers [13–21] by using a finite element method and Isisa and his coworkers [21–24] by using a force method.

2. Numerical results and discussions

In order to study the effect of the geometry of the surface cavity on the stress intensity factors, first of all, we should pay our attentions to surface semi-circular crack to find out when we can treat the cracked body as infinite. According to Newman and his coworkers' research [13], the following modes are considered:

(1) $R/t = 0.1$, $R/W = 0.1$ and $R/H = 0.1$.
(2) $R/t = 0.2$, $R/W = 0.2$ and $R/H = 0.2$.

The calculated SIFs normalized by $\sigma\sqrt{(\pi a)/Q}$ by using FRANC3D are given in Table 1. For comparison purpose, Table 1 also lists those reported in Ref. [13], from which it can be seen that the agreement is very good, except at $2\phi/\pi = 0$ (where the crack front curve intersects the free surface) at which the biggest error exists. This error should be due to the presupposition of plane strain.

From Table 1, we can see that the maximum SIFs occur at $2\phi/\pi = 0$ and we can easily come to the conclusion that if $R/t \leqslant 0.2$, $R/W \leqslant 0.2$ and $R/H \leqslant 0.2$ we can accept the cracked body as infinite. For convenience, we denote the maximum SIFs in mode (2) as K_{norm} ($K_{\text{norm}} = 1.251$).

And then we can study our interested mode (see Fig. 1). As done for the above study of the surface semi-circular crack, we should also learn when we ignore the boundary effect on the SIFs. The following modes are considered:

(1) $R/t = 0.125$, $R/W = 0.2$, $R/H = 0.2$ and $R/r = 2$.
(2) $R/t = 0.2$, $R/W = 0.125$, $R/H = 0.125$ and $R/r = 2$.
(3) $R/t = 0.2$, $R/W = 0.2$, $R/H = 0.2$ and $R/r = 2$.

For convenience, we denote the SIFs normalized by $\sigma\sqrt{(\pi a)/Q}$ by K_{IN} when we study the crack emanating from a surface semi-spherical cavity in a infinite elastic body. Then the K_{IN} calculated by FRANC3D are given in Table 2, from which it can be seen that the maximum SIFs also occur at $2\phi/\pi = 0$ and it is concluded that a cracked body with $R/t \leqslant 0.2$, $R/W \leqslant 0.2$ and $R/H \leqslant 0.2$ can be perceived as infinite.

In the last, the following modes are selected to reveal the effect of the geometry of the surface cavity on the stress intensity factors (SIFs).

$$R/r = 1.2, 1.24, 1.4, 1.6, 1.8, 2, 3, 4, 5.5$$

The calculated maximum K_{IN} (denoted by K_{INm}) for each mode by using FRANC3D are given in Table 3. For the purpose of conveniently observing the effect of the geometry of the surface cavity on the SIFs, the data in Table 3 are pictured in Fig. 2. If we denote R/r by a ($a \succ 1$), the R/r when $K_{\text{INm}} = K_{\text{norm}}$ by a_1($a_1 \approx 1.351$), $R/r = 2$ by a_2 (see Fig. 2). Thus K_{INm} is only a function of a.Then it is concluded that:

Table 1
The calculated SIFs for semi-circular crack the of the two selected modes

$2\phi/\pi$	Mode (1)	Mode (2)	Error (%) (mode (1) – mode (2))/mode (1)	Newman	Error (%) (mode (2) – Newman)/Newman
0	1.231	1.251	1.61	1.174	6.56
0.125	1.164	1.185	1.82	1.145	3.49
0.25	1.115	1.137	1.92	1.105	2.85
0.375	1.082	1.101	1.78	1.082	1.79
0.5	1.064	1.082	1.69	1.067	1.37
0.625	1.059	1.077	1.64	1.058	1.75
0.75	1.064	1.081	1.65	1.053	2.68
0.875	1.068	1.084	1.45	1.050	3.24
1	1.059	1.075	1.51	1.049	2.51

Table 2
The calculated K_{IN} of the three selected modes

$2\phi/\pi$	Mode (1)	Mode (2)	Mode (3)	Error (%) (mode (3) – mode (1))/mode (1)	Error (%) (mode (3) – mode (2))/mode (2)
0	1.378	1.366	1.383	0.351	1.222
0.125	1.223	1.213	1.224	0.159	0.912
0.25	1.178	1.169	1.180	0.160	0.951
0.375	1.161	1.152	1.163	0.167	0.951
0.5	1.152	1.143	1.154	0.167	0.920
0.625	1.145	1.137	1.147	0.163	0.888
0.75	1.139	1.131	1.141	0.159	0.863
0.875	1.135	1.127	1.136	0.153	0.846
1	1.131	1.123	1.133	0.150	0.839

Table 3
K_{INm} calculated by FRANC3D for the considered modes

a	1.2	1.24	1.4	1.6	1.8	2.0	2.5	3.0	4.0	5.5
K_{INm}	1.143	1.186	1.280	1.342	1.367	1.383	1.317	1.289	1.269	1.263

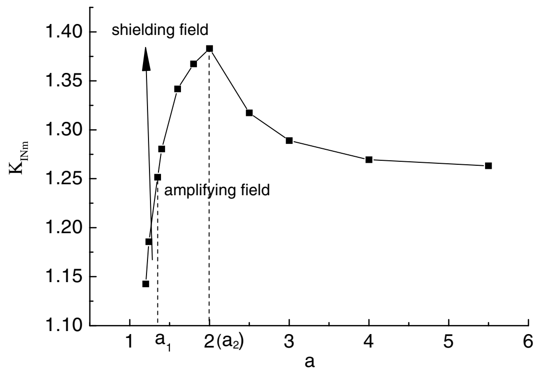

Fig. 2. Variation of normalized SIFs with a.

(1) K_{INm} increases with increase of a when $a \leqslant a_2$. And the biggest of K_{INm} occurs at about a_2.
(2) When $a \succ a_2$, K_{INm} decreases with increase of a. When $a = 5.5$, K_{INm} is 1.010 times of K_{norm}, which illustrate the effect of the surface cavity on the SIFs can be neglected.
(3) When $a \leqslant a_1$, K_{INm} is less than K_{norm}, which illustrates the surface cavity has a shielding influence on the surface crack.
(4) When $a \succ a_1$, the surface cavity has the amplifying influence on the SIFs. When $a = a_2$, it amplifies the SIFs by 10.5%.

3. Conclusions

(1) It is found that there is such a dimensionlessa_1 that as a is less than a_1 the surface cavity has a shielding influence on the surface crack and that while a is more than a_1 the surface cavity has the amplifying influence on the SIFs.
(2) The biggest amplifying factor is about 10.5%.
(3) The amplifying influence of the surface cavity on the SIFs is neglectable as $a > 4$.

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